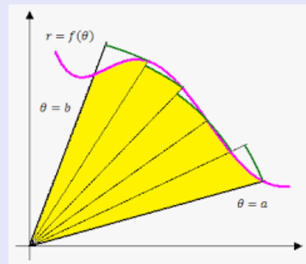
Calculus II

Lecture 8



Feb 19-8:47 AM

Class QZ 7

Use integration by Parts to evaluate $\int x^2 \ln x \, dx$.

$u = \ln x \qquad dv = x^2 dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^3}{3}$

$$uv - \int v\,du = \frac{x^3}{3}\ln x - \int \frac{x^3}{3} \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C$$

$$= \boxed{\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C}$$

$$\int_1^e x^2 \ln x \, dx$$

$$= \left(\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3\right)\Big|_1^e$$

$$= \frac{1}{3} \cdot e^3 \cdot \cancelto{1}{\ln e} - \frac{1}{9} \cdot e^3 - \frac{1}{3} \cdot 1^3 \cdot \cancelto{0}{\ln 1} + \frac{1}{9} \cdot 1^3$$

$$= \frac{3}{9}e^3 - \frac{1}{9}e^3 - 0 + \frac{1}{9}$$

$$= \boxed{\frac{2e^3}{9} + \frac{1}{9}} = \boxed{\frac{2e^3 + 1}{9}}$$

Jun 24-6:48 AM

Evaluate

$$\int (\ln x)^n dx = \int (\ln x)^{n-1} \ln x \, dx$$

$$= x(\ln x)^n - \int \cancel{x}\, n(\ln x)^{n-1} \cdot \frac{1}{\cancel{x}} dx$$

$u = (\ln x)^n \qquad dv = dx$

$du = n(\ln x)^{n-1} \cdot \frac{1}{x} \qquad v = x$

$$= \boxed{x(\ln x)^n - n\int (\ln x)^{n-1} dx}$$

we want to show

$$\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1} dx$$

$$\int (\ln x)^3 dx = x(\ln x)^3 - 3\int (\ln x)^2 dx$$

$$= x(\ln x)^3 - 3\left[x(\ln x)^2 - 2\int (\ln x)^1 dx\right]$$

$$= x(\ln x)^3 - 3x(\ln x)^2 + 6\int \ln x \, dx$$

$$= \boxed{x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x + C}$$

$$\int \ln x \, dx = x\ln x - \int 1 dx = x\ln x - x + C$$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x} dx \qquad v = x$

Jun 24-8:17 AM

Show $\int x^n e^x dx = x^n e^x - n\int x^{n-1} e^x dx$

$u = x^n \qquad dv = e^x dx$

$du = nx^{n-1} dx \qquad v = e^x$

$$= x^n e^x - \int n x^{n-1} e^x dx$$

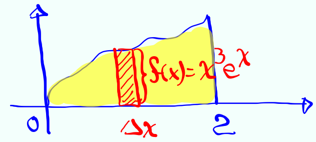
$$= x^n e^x - n\int e^{n-1} e^x dx$$

Jun 24-8:31 AM

Find the area below $f(x)=x^3e^x$, above x-axis from $x=0$ to $x=2$. Detailed drawing required.

$f(0)=0$

$f(2)=8e^2$



$A=\int_0^2 x^3e^x\,dx$

$\int x^n e^x dx = x^n e^x - n\int x^{n-1}e^x dx$

$= x^3e^x - 3\int x^2e^x dx$

$= x^3e^x - 3\left[x^2e^x - 2\int x^1 e^x dx\right]$

$u=x \quad dv=e^x dx$

$du=dx \quad v=e^x$

$= x^3e^x - 3x^2e^x + 6\int x e^x dx$

$= \left(x^3e^x - 3x^2e^x + 6xe^x - 6e^x\right)\Big|_0^2$

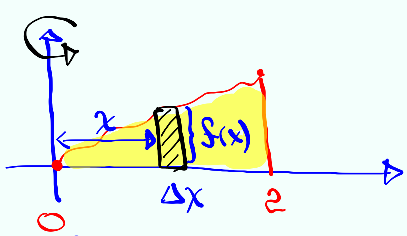
$= 8e^2 - \cancel{12e^2} + \cancel{12e^2} - 6e^2 + 6e^0$

$= \boxed{2e^2+6}$

Jun 24-8:36 AM

Rotate the region bounded by $f(x)=x^2e^x$, $y=0$, $x=0$, and $x=2$ by the Y-axis. Find its volume. Drawing Required.

$f(0)=0$, $f(2)=4e^2$



Shell

$V=\int_0^2 2\pi\cdot x\cdot x^2e^x\,dx = 2\pi\boxed{\int_0^2 x^3e^x dx}$

See last example

$=2\pi\left[2e^2+6\right] = 4\pi\left[e^2+3\right]$

Jun 24-8:45 AM

Show $\int \sec^n x\, dx = \frac{1}{n-1} \tan x \sec^{n-2} x + \frac{n-2}{n-1} \int \sec^{n-2} x\, dx$

Hint:

$\sec^n x = \sec^{n-2} x \cdot \sec^2 x$

$\int \sec^n x = \int \sec^{n-2} x \cdot \sec^2 x\, dx$

$u = \sec^{n-2} x$ $\quad dV = \sec^2 x\, dx$

$du = (n-2)\sec^{n-3} \cdot \sec x \tan x\, dx$ $\quad V = \tan x$

$= \tan x \sec^{n-2} x - \int (n-2) \sec^{n-2} \tan^2 x\, dx$

$= \tan x \sec^{n-2} x - (n-2) \int \sec^{n-2} x \cdot (\sec^2 x - 1) \cdot dx$

$= \tan x \sec^{n-2} x \boxed{- (n-2) \boxed{\int \sec^n x}} + (n-2) \int \sec^{n-2} x\, dx$

$1 \int \sec^n x\, dx + (n-2) \int \sec^n x\, dx = \tan x \sec^{n-2} x + (n-2) \int \sec^{n-2} dx$

$(n-1) \int \sec^n x\, dx = \tan x \sec^{n-2} x + (n-2) \int \sec^{n-2} x\, dx$

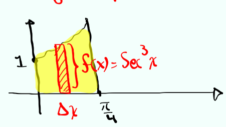
$\int \sec^n x = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x\, dx$

$n \neq 1$

Jun 24-8:52 AM

Find the area below $f(x) = \sec^3 x$, above x-axis from $x = 0$ to $x = \frac{\pi}{4}$.

Drawing required.



$f(0) = \sec^3 0 = 1^3 = 1$

$f(\frac{\pi}{4}) = \sec^3 \frac{\pi}{4} = \frac{1}{\cos^3 \frac{\pi}{4}} = \frac{1}{(\frac{\sqrt{2}}{2})^3} = \frac{8}{2\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

$A = \int_0^{\pi/4} \sec^3 x\, dx$

$u = \sec x$ $\quad dV = \sec^2 x\, dx$

$du = \sec x \tan x\, dx$ $\quad V = \tan x$

$\int \sec^3 x\, dx = \sec x \tan x - \int \sec x \tan^2 x\, dx$

$= \sec x \tan x - \int \sec x (\sec^2 x - 1)\, dx$

$= \sec x \tan x - \int \sec^3 x\, dx + \int \sec x\, dx$

$2 \int \sec^3 x\, dx = \sec x \tan x + \int \sec x\, dx$

$2 \int \sec^3 x\, dx = \sec x \tan x + \ln|\sec x + \tan x| + C$

$\int \sec^3 x\, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C$

$A = \frac{1}{2} \Big[\sec \frac{\pi}{4} \tan \frac{\pi}{4} + \ln(\sec \frac{\pi}{4} + \tan \frac{\pi}{4}) - \sec 0 \tan 0 - \ln|\sec 0 + \tan 0| \Big]$

$\boxed{A = \frac{1}{2} [\sqrt{2} + \ln(\sqrt{2} + 1)]}$

Jun 24-9:06 AM

Recall

$\int \cos x \, dx = \sin x + C$

$\int \sin x \, dx = -\cos x + C$

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1 \Rightarrow \cos^2 x = \dfrac{1 + \cos 2x}{2}$

$\cos 2x = 1 - 2\sin^2 x \Rightarrow \sin^2 x = \dfrac{1 - \cos 2x}{2}$

$\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$

$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$

$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$

Jun 24-9:47 AM

$\int \sin^3 x \cos^4 x \, dx$

$= \int \sin^2 x \cos^4 x \sin x \, dx$

$= \int (1 - \cos^2 x) \cdot \cos^4 x \cdot \sin x \, dx$

$= \int (1 - u^2) u^4 \cdot -du$

$= \int (u^6 - u^4) du = \frac{1}{7}u^7 - \frac{1}{5}u^5 + C$

$= \boxed{\frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + C}$

$\sin^2 x + \cos^2 x = 1$

$\frac{d}{dx}[\cos x] = -\sin x$

$\frac{d}{dx}[\sin x] = \cos x$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

Jun 24-9:53 AM

$$\int_0^{\frac{\pi}{2}} \cos^5 x\,dx \qquad \cos^5 x = \cos^4 x \cdot \cos x$$

$$= \left[\cos^2 x\right]^2 \cdot \cos x$$

$$= \int_0^{\pi/2} \left[1 - \sin^2 x\right]^2 \cos x\,dx \qquad = \left[1 - \sin^2 x\right]^2 \cdot \cos x$$

$u = \sin x \qquad du = \cos x\,dx$

$x = 0 \rightarrow u = 0$

$$= \int_0^1 (1 - u^2)^2\,du \qquad x = \frac{\pi}{2} \rightarrow u = 1$$

$$= \int_0^1 \left[1 - 2u^2 + u^4\right] du = \left(u - \frac{2u^3}{3} + \frac{u^5}{5}\right)\Big|_0^1$$

$$= \boxed{\frac{8}{15}}$$

Jun 24-9:58 AM

$$\int \sin 8x \cos 4x\,dx$$

use

$$\sin A \cos B = \frac{1}{2}\left[\sin(A-B) + \sin(A+B)\right]$$

$$= \int \frac{1}{2}\left[\sin 4x + \sin 12x\right] dx$$

$$= \frac{1}{2} \cdot \left[-\frac{\cos 4x}{4} + \frac{-\cos 12x}{12}\right] + C$$

Jun 24-10:04 AM

$$\int_0^1 \cos\pi x \cos 5\pi x \, dx$$

Recall

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$= \int_0^1 \frac{1}{2}\left[\cos(\overbrace{-4\pi x}^{4\pi x}) + \cos(6\pi x)\right]dx \qquad \cos(-\alpha) = \cos\alpha$$

$$= \frac{1}{2}\left[\frac{\sin 4\pi x}{4\pi} + \frac{\sin 6\pi x}{6\pi}\right]\Bigg|_0^1 = \boxed{0}$$

Do not use $\emptyset$ for Zero.

Jun 24-10:07 AM

Find f_{ave} for $f(x) = \sin^2 x \cos^4 x$ on $[0, \frac{\pi}{2}]$

$$f_{ave} = \frac{1}{\frac{\pi}{2} - 0}\int_0^{\pi/2} \sin^2 x \cos^4 x \, dx$$

$$= \frac{2}{\pi}\int_0^{\pi/2} [\cos^4 x - \cos^6 x]dx \qquad \text{Recall } \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\cos^4 x = [\cos^2 x]^2 = \left[\frac{1 + \cos 2x}{2}\right]^2 = \frac{1 + 2\cos 2x + \cos^2 2x}{4}$$

$$\int \cos^4 x \, dx = \int\left(\frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{4}\cos^2 2x\right)dx$$

$$= \frac{1}{4}x + \frac{1}{2}\cdot\frac{\sin 2x}{2} + \frac{1}{4}\int \cos^2 2x \, dx$$

$$= \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{4}\int\frac{1 + \cos 4x}{2}dx$$

to be completed by you.

$$= \frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}\left[x + \frac{\sin 4x}{4}\right]\Bigg|_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\cdot\frac{\pi}{2} + \frac{1}{8}\left[\frac{\pi}{2}\right] = \frac{\pi}{8} + \frac{\pi}{16} = \boxed{\frac{3\pi}{16}}$$

Jun 24-10:12 AM

$$\int x \sin^3 x \, dx \qquad u = x \qquad dV = \sin^3 x$$

$$du = dx \qquad V = -\cos x + \frac{1}{3}\cos^3 x$$

$$V = \int \sin^3 x \, dx = \int \sin^2 x \cdot \sin x \, dx = \int (1 - \cos^2 x) \cdot \sin x \, dx$$

$$= \int \sin x \, dx - \int \cos^2 x \sin x \, dx$$

$$= -\cos x + \frac{1}{3}\cos^3 x$$

$$\int x \sin^3 x \, dx = -x\left(\cos x - \frac{1}{3}\cos^3 x\right) - \int \left(-\cos x + \frac{1}{3}\cos^3 x\right) dx$$

$$= -x\cos x + \frac{1}{3}x\cos^3 x + \int \cos x \, dx - \frac{1}{3}\int \cos^3 x \, dx$$

$$= -x\cos x + \frac{1}{3}x\cos^3 x + \sin x - \frac{1}{3}\int \cos^3 x \, dx$$

Jun 24-10:24 AM

$$\int \cos^3 x \, dx = \int \cos^2 x \cdot \cos x \, dx$$

$$= \int (1 - \sin^2 x) \cdot \cos x \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$= \sin x - \frac{1}{3}\sin^3 x$$

Jun 24-10:33 AM

$$\int_0^{\pi/4} \sec^4 x \ \tan^4 x \ dx$$

$\sec^2 x = 1 + \tan^2 x$

$\frac{d}{dx}[\tan x] = \sec^2 x$

$$= \int_0^{\pi/4} \sec^2 x \cdot \tan^4 x \cdot \sec^2 x \, dx$$

$u = \tan x$

$du = \sec^2 x \, dx$

$$= \int_0^1 (1 + u^2) \cdot u^4 \cdot du$$

$x = 0 \rightarrow u = 0$

$x = \frac{\pi}{4} \rightarrow u = 1$

$$= \left(\frac{u^5}{5} + \frac{u^7}{7}\right)\Bigg|_0^1 = \boxed{\frac{12}{35}}$$

Jun 24-10:39 AM

$$\int x \sec x \tan x \ dx$$

$u = x$ $\quad dV = \sec x \tan x \, dx$

$du = dx$ $\quad V = \sec x$

$$= x \sec x - \int \sec x \, dx$$

$$= x \sec x - \ln|\sec x + \tan x| + C$$

Jun 24-10:44 AM

$$\int \tan^5 x \sec^3 x \, dx$$

Hint: Make exponents to become even.

$$= \int \tan^4 x \sec^2 x \cdot \sec x \tan x \, dx$$

$$= \int [\underbrace{\tan^2 x}_{(\sec^2 x - 1)^2}]^2 \sec^2 x \cdot \sec x \tan x \, dx$$

Let $u = \sec x$

$du = \sec x \tan x \, dx$

$$= \int (u^2 - 1)^2 \cdot u^2 \cdot du =$$

$$= \int (u^6 - 2u^4 + u^2) \, du$$

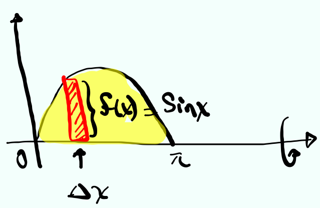
$$= \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} + C$$

$$= \frac{\sec^7 x}{7} - \frac{2\sec^5 x}{5} + \frac{\sec^3 x}{3} + C$$

Jun 24-10:47 AM

Class QZ 8 (open Notes)

Find the volume by rotating the region bounded by $y = \sin x$, $y = 0$, $x = 0$, $x = \pi$ about x-axis. Drawing required.



Disk Method

$$V = \int_0^{\pi} \pi [\sin x]^2 \, dx$$

$$= \pi \cdot \int_0^{\pi} \sin^2 x \, dx =$$

$$= \pi \cdot \int_0^{\pi} \frac{1 - \cos 2x}{2} \, dx$$

$$= \frac{\pi}{2} \left[x - \frac{\sin 2x}{2} \right] \Big|_0^{\pi} \quad = \boxed{\frac{\pi^2}{2}}$$

Jun 24-10:54 AM